An analytical approach to the coupled carbon-climate-human system

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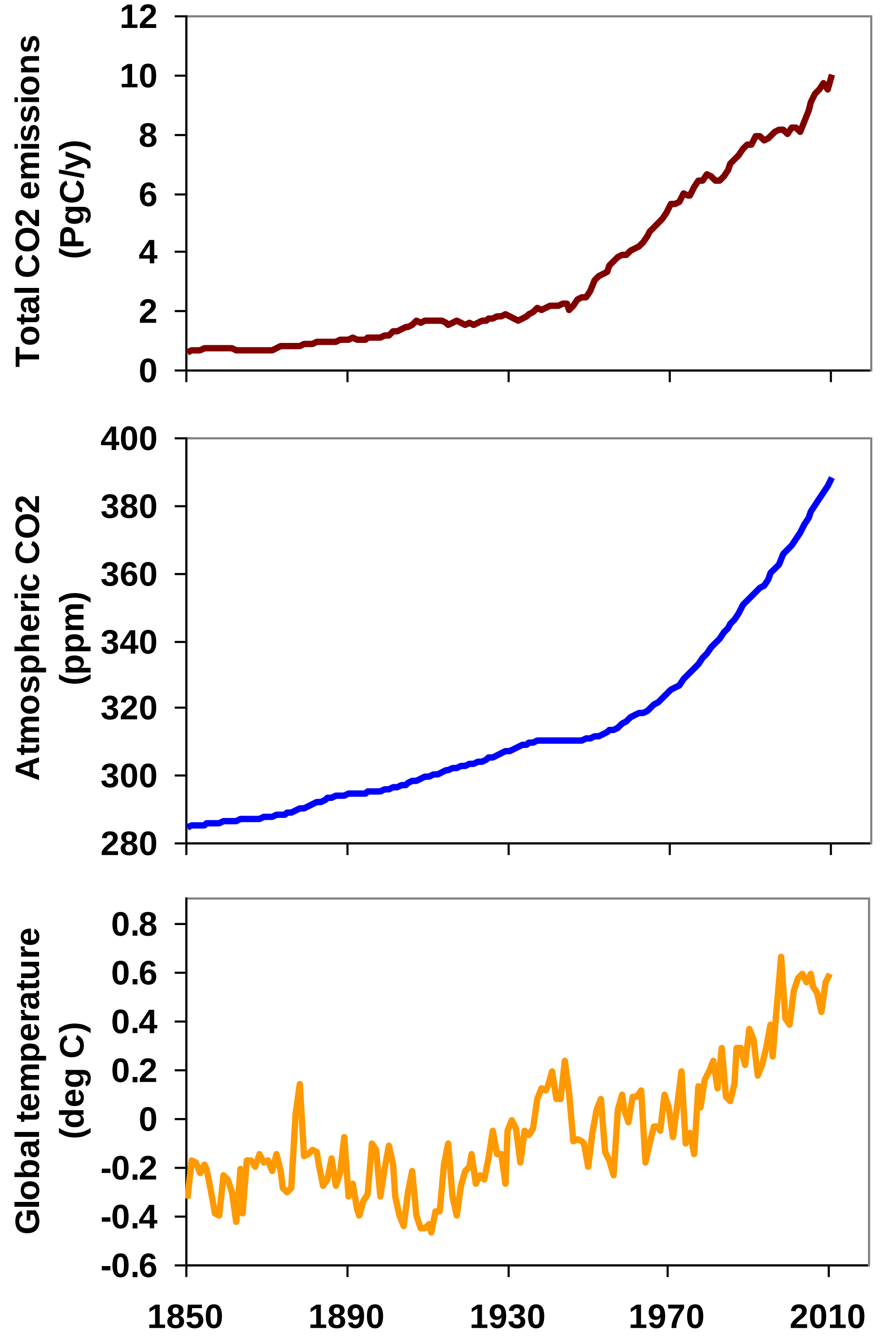
CSIRO

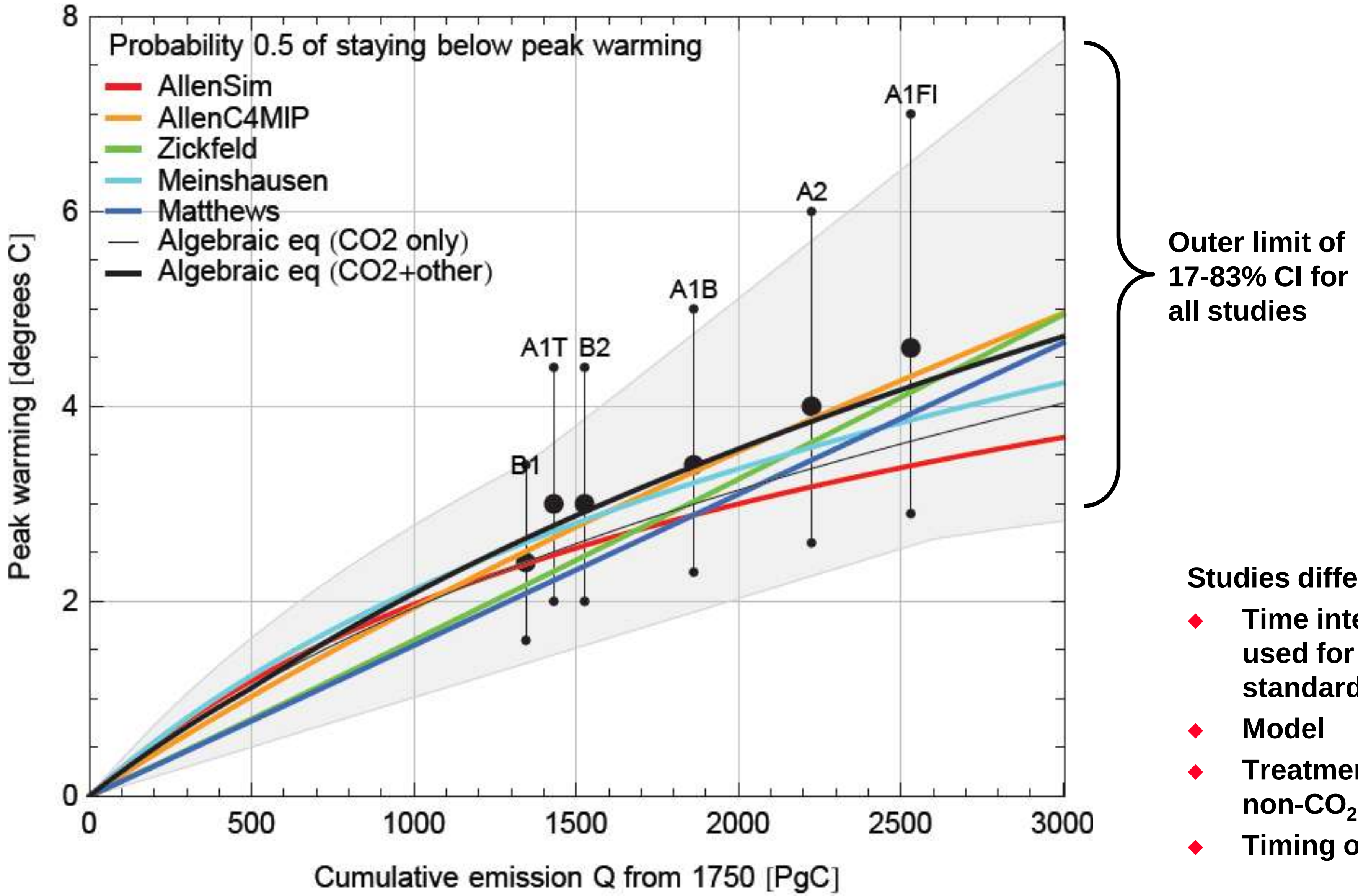
Global Carbon Project

Earth system: forcing and responses

- CO_2 emissions (fossil fuels + land use change)
- CO_2 concentrations (composite record)
- Global temperature (land + ocean, HadCRU)



Warming as a function of cumulative CO_2 emissions Synthesis of 5 model studies and IPCC (2007)



Studies differ in:

- **Time interval used for Q (here standardised)**
- **Model**
- **Treatment of non-CO_2 forcing**
- **Timing of T(Q)**

Raupach et al. (2011) CAWCR Report 42

Ratios between fluxes and stores

- [Atmospheric CO_2 budget]

$$\underbrace{dc_A/dt}_{\text{CO}_2\text{ growth rate}} = \underbrace{f_E}_{\text{FF + LUC}} - \underbrace{f_S}_{\text{Land + Ocean}}$$

- Airborne fraction:

$$\text{AF} = \left[\frac{\text{CO}_2\text{ growth rate}}{\text{Emissions}}\right] = \frac{dc_A/dt}{f_E}$$

- Cumulative AF:

$$\text{CAF} = \left[\frac{\text{Excess CO}_2}{\text{Cumulative emissions}}\right] = \frac{c_A}{Q}$$

- Sink rate [1/y]:

$$k_S = \left[\begin{matrix}\text{CO}_2\text{ uptake rate}\\ \text{per unit excess CO}_2\end{matrix}\right] = \frac{f_S}{c_A}$$

- T/Q ratio:

$$\left[\frac{\text{Excess temperature}}{\text{Cumulative emissions}}\right] = \frac{T}{Q}$$

AF, CAF, sink rate

- CO_2 Airborne Fraction
  Cumulative AF
  - near constant?

- CO_2 sink rate (k_S)
  - declining

$$\underbrace{dc_A/dt}_{\text{CO}_2\text{ growth rate}} = \underbrace{f_E}_{\text{FF + LUC}} - \underbrace{f_S}_{\text{Land + Ocean}}$$

$$\text{AF} = \left[\frac{\text{CO}_2 \text{ growth rate}}{\text{Emissions}}\right] = \frac{dc_A/dt}{f_E}$$

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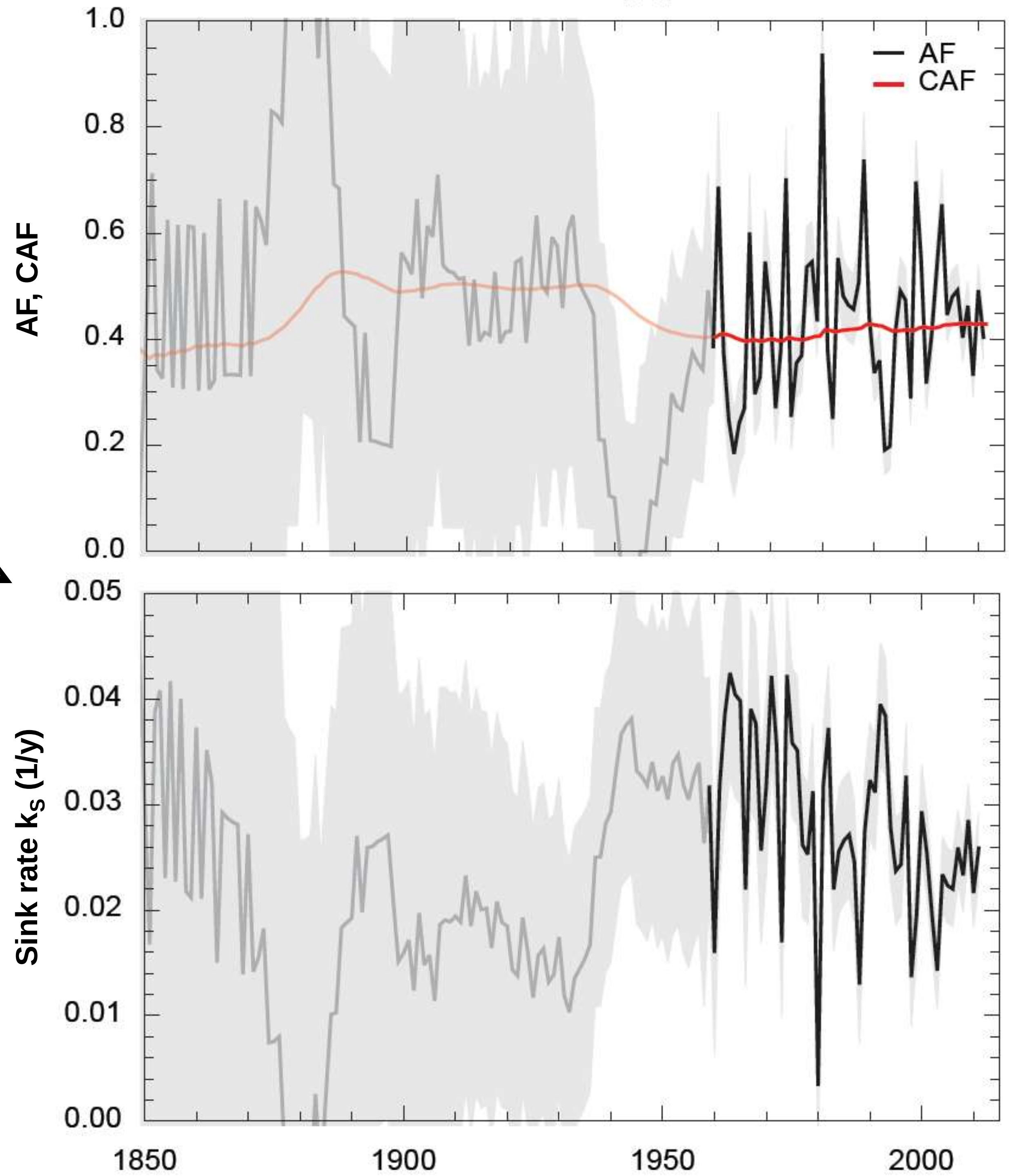
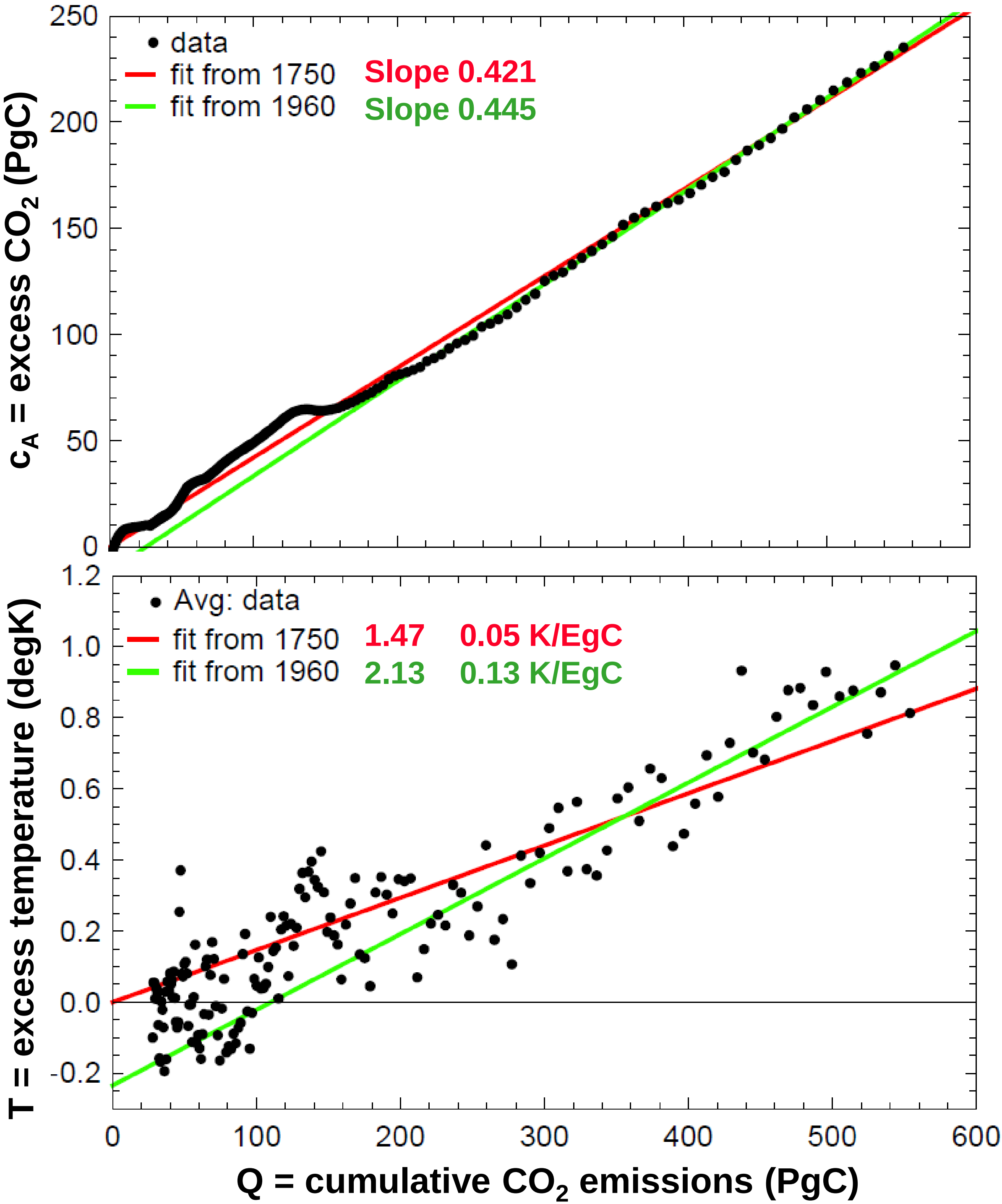


CO_2 and T
Past data

- c_A = excess CO_2 (PgC)
  $c_A = 2.13\ (CO_2 - 280\text{ppm})$
- T = excess temperature (ref 1880-1900)
- Q(t) = cumulative CO_2 emissions from 1750
- Plot c_A and T against cumulative CO_2 emissions Q(t)



Questions

- PAST
  - Why are ratios among fluxes and stores (AF, CAF, T/Q) near constant from ~1850 to present, in the face of a 20-fold increase in emissions?
  - To the extent that these ratios have changed, why so?

- FUTURE
  - How will AF, CAF and T/Q behave in future?
    - In particular, do we expect continuance of a near-proportional relationship between T and Q?
    - If so, why?

Linear theory

- Linearise carbon-climate system (deal with nonlinearities later)
- State variables = (carbon pools, temperatures, other gases …)
  - Dimension (number of state variables) can be as high as we want (10 or 10^7)
- Linear theory makes available a rich set of analytic resources:
  - Normal modes, Green's functions, transforms (Laplace, Fourier, …)
- Linear theory provides complementary insights to numerical modelling
- Ways in which linear theory is embedded in climate science:
  - Pulse response functions for CO_2 (ocean mixed layer, atmosphere)
  - Step response functions for climate
  - CO_2 equivalence and Global Warming Potentials

Linear theory

- Nonlinear system:

$$\frac{d\mathbf{x}(t)}{dt} = \underbrace{\mathbf{f}(t)}_{\text{Forcing}} + \underbrace{\boldsymbol{\Phi}(\mathbf{x})}_{\text{Response}}$$

- Linearised system:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(t) - \mathbf{K}\mathbf{x}$$

$$\mathbf{K} = -\frac{\partial \boldsymbol{\Phi}}{\partial \mathbf{x}}$$

$$\frac{d\,[\text{state variables}]}{dt} = \text{forcing} - \begin{bmatrix}\text{response}\\ \text{matrix}\end{bmatrix}\begin{bmatrix}\text{state}\\ \text{variables}\end{bmatrix}$$

- $\mathbf{x}(t)$ = vector of state variables (carbon pools, temperatures)
- $\mathbf{f}(t)$ = vector of external forcing fluxes
- $\boldsymbol{\Phi}(\mathbf{x})$ = vector of response fluxes
- $\mathbf{K}$ = linear response matrix = −[Jacobian of $\boldsymbol{\Phi}(\mathbf{x})$]

Insights from linear theory

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(t) - \mathbf{K}\mathbf{x}$$

- **Basic fact:** for a linear system (Lin):
  - any exponential function of time is an **eigenfunction** of the system
  - [eigenfunction: forcing and response have the same shape]
- **Theorem:** for a linear system (Lin) with exponential forcing (Exp):
  - All state variables grow at forcing rates (not response rates)
  - All ratios among state variables and fluxes approach constant values
  - These ratios "forget" initial state at forcing rates (not response rates)
- For the carbon-climate system in the LinExp idealisation, we would have
  - c_{Air}/Q = constant
    c_{Land}/Q = constant
    c_{Ocean}/Q = constant
  - AF = CAF = constant
  - Sink rate k_S = constant
  - T/Q = constant

- c_{Air} = anthropogenic C in atmosphere
- c_{Land} = anthropogenic C in land stores
- c_{Ocean} = anthropogenic C in ocean stores
- Q = cumulative anthropogenic C emissions
- T = perturbation temperature

Nonlinear carbon-climate model

- SCCM = Simple Carbon-Climate Model
  - Raupach et al (2011) Tellus
  - Harman, Trudinger, Raupach (2011) CAWCR Report

- State vector $(C_A, C_{B1}, C_{B2}, C_{M1}, C_{M2}, C_{M3}, C_{M4}, C_{MD}, nonCO_2, T_{M1}, T_{M2}, T_{M3})$
  - C_A: **1 atmos C pool**
  - C_{B1}, C_{B2}: **2 land C pools**
  - $C_{M1}, C_{M2}, C_{M3}, C_{M4}, C_{MD}$: **5 ocean C pools**
  - $nonCO_2$: **4 non-CO2 GHGs**
  - T_{M1}, T_{M2}, T_{M3}: **3 temperature pools**

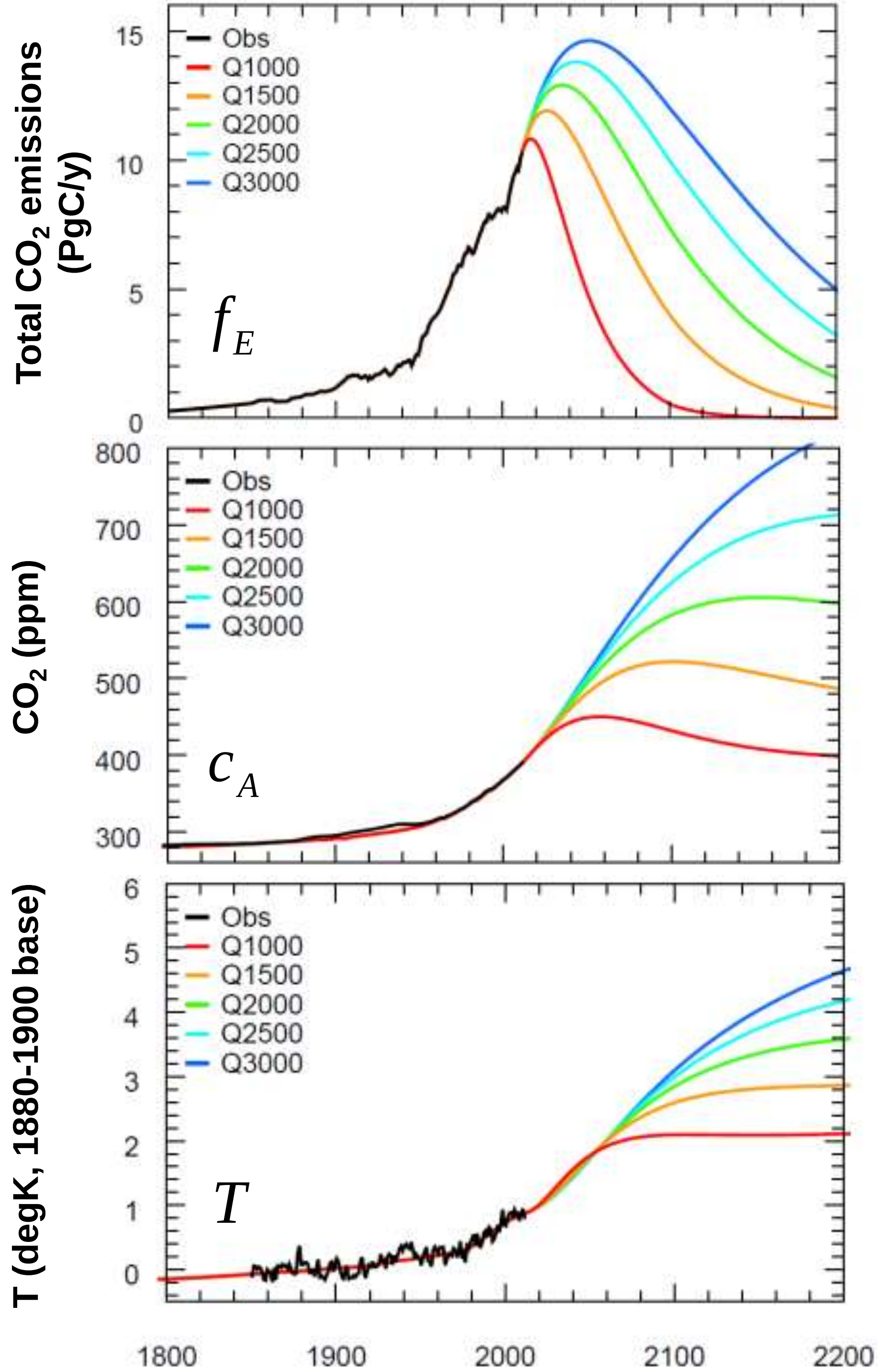
- Nonlinearities: Radiative forcing is nonlinear in gas concentrations
  Land and ocean CO_2 fluxes are nonlinear in CO_2, temp
  Volcanic influence on terrestrial NPP

SCCM results: Vary cumulative emissions

- Plots against time

- Full model
- All forcings (CO_2, CH_4, N_2O, CFCs, aerosols)
- Aerosol RF ~ f_{Foss} (tech factor)
- Analytic scenarios for future emissions trajectories
- All-time cumulative cap on CO_2 emissions Q = 1000 to 3000 PgC

$$Q(t) = \int_{1750}^{t} f_E(\tau)\, d\tau$$



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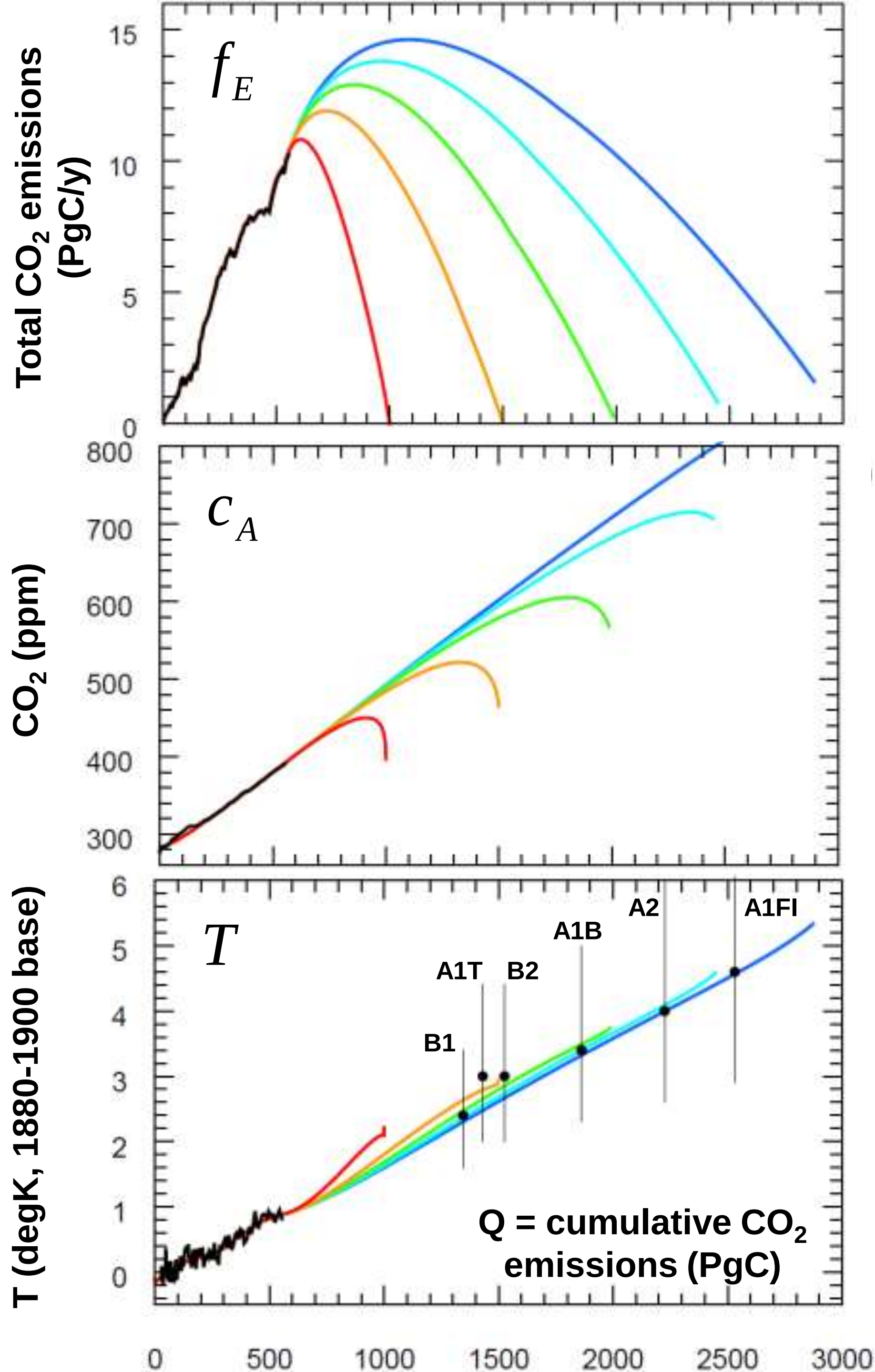


CO_2 and T Attribution of trends

Progressive simplification:

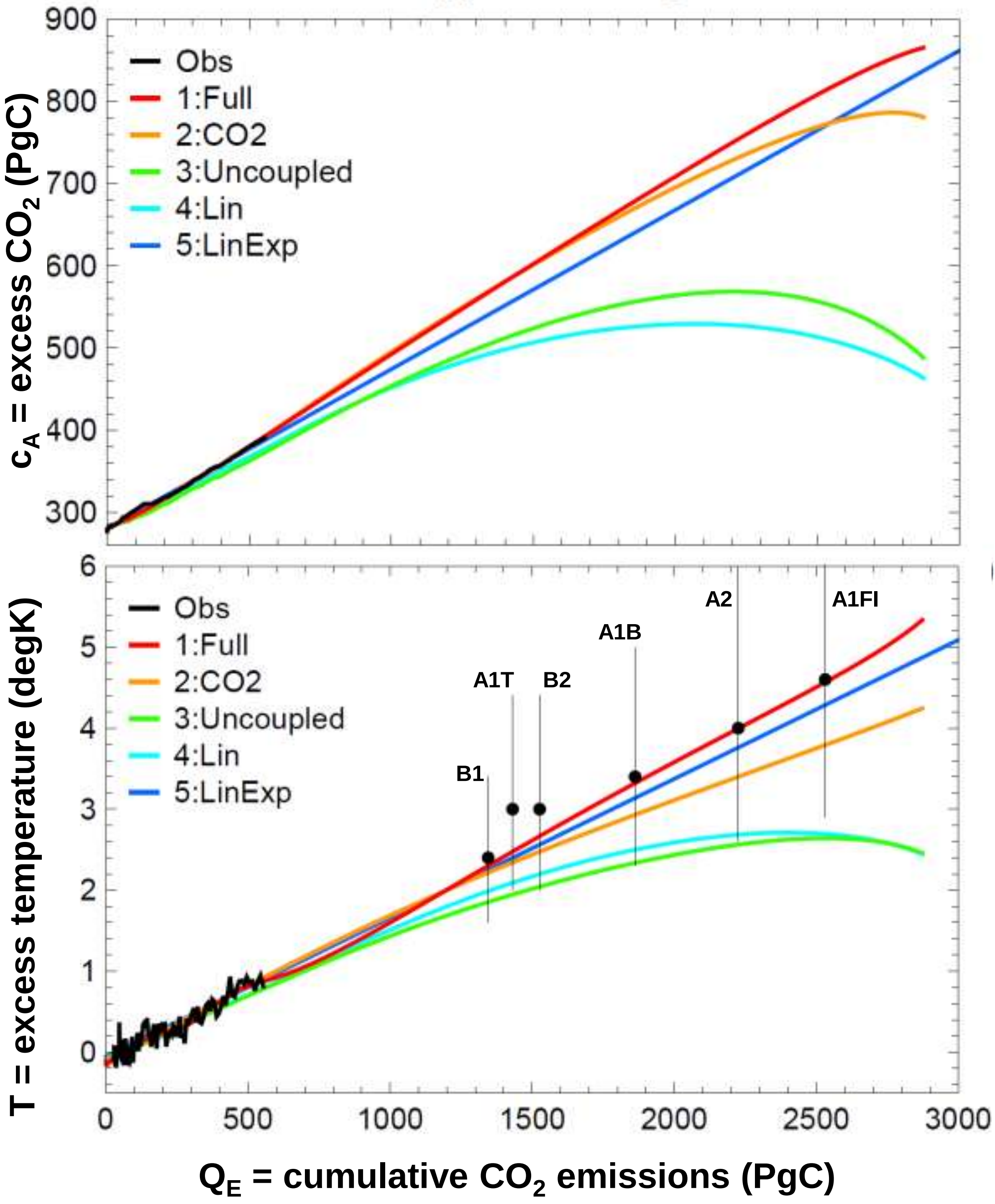
1: **Full model**

2: **CO_2 only**
(remove non-CO_2 forcing)

3: **Uncoupled**
(remove dependence of CO_2 sink fluxes on temperature)

4: **Linearised**
(remove nonlinearities in CO_2 fluxes and radiative forcing)

5: **LinExp**
(impose exponential emissions flux)

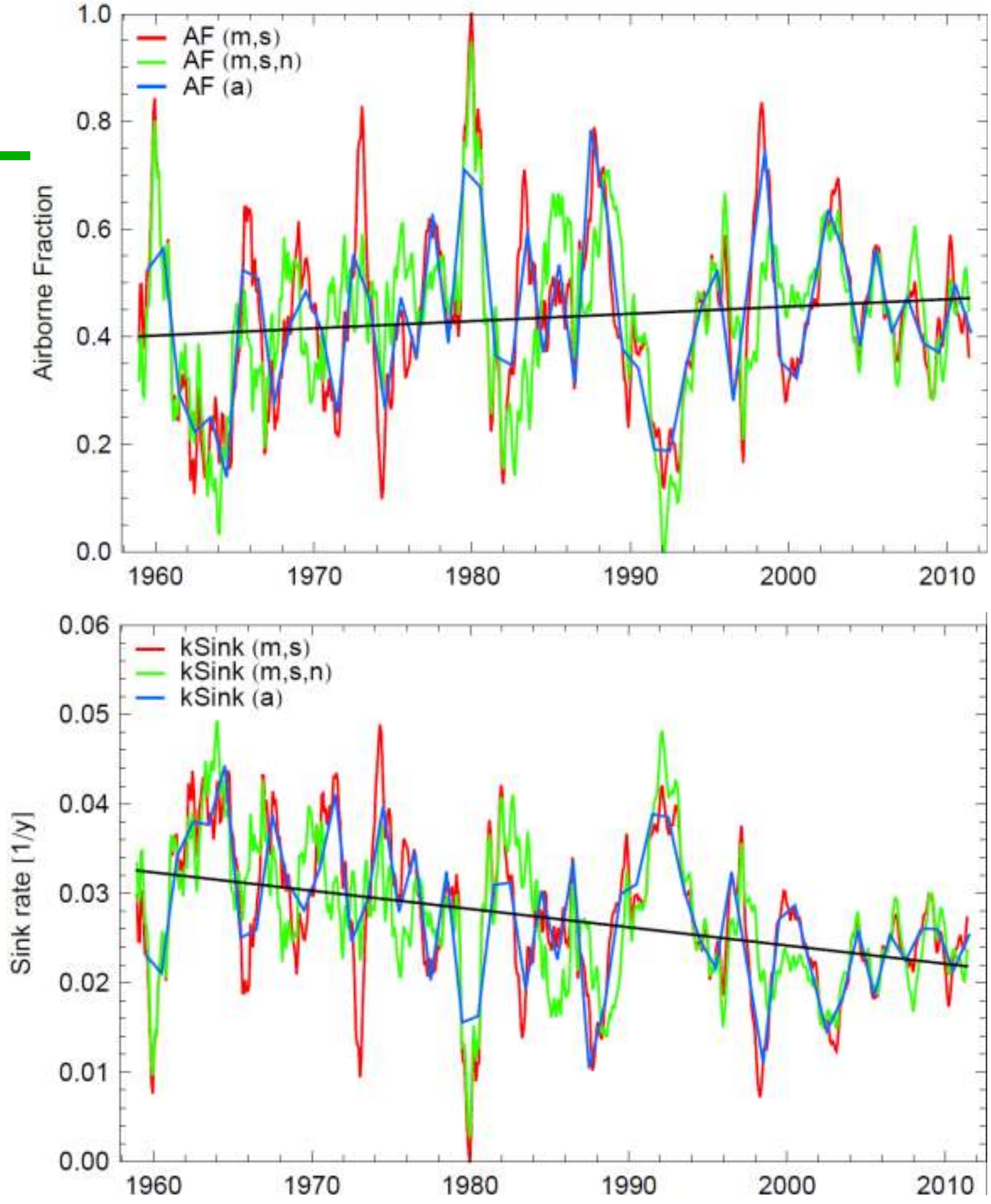


Past AF and sink rate

- Airborne fraction:
  Proportional growth rate of AF = **0.3 % y^{-1}** (P=0.8)
  Canadell et al (2007)
  Raupach et al (2008)
  Le Quere et al (2009)

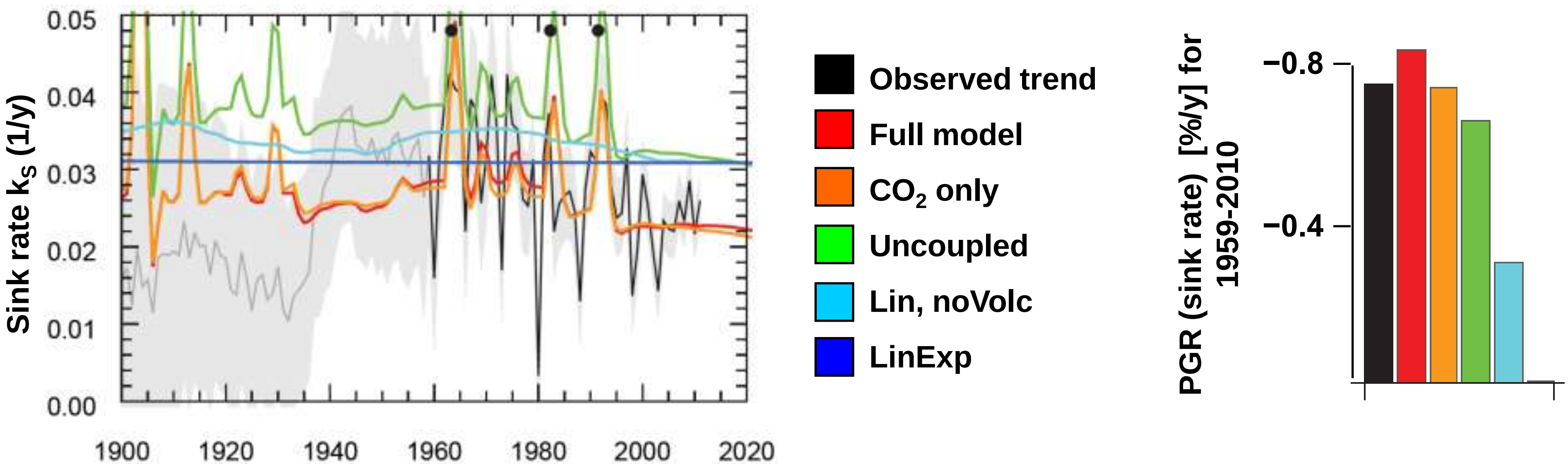
  Trend, attribution contested:
  * Knorr (2009)
  * Gloor et el (2010)
  * Sarmiento et al (2010)
  * Francey (2010)

- CO_2 sink rate:
  Proportional growth rate of k_S = **−0.8 % y^{-1}**
  (P=0.998)



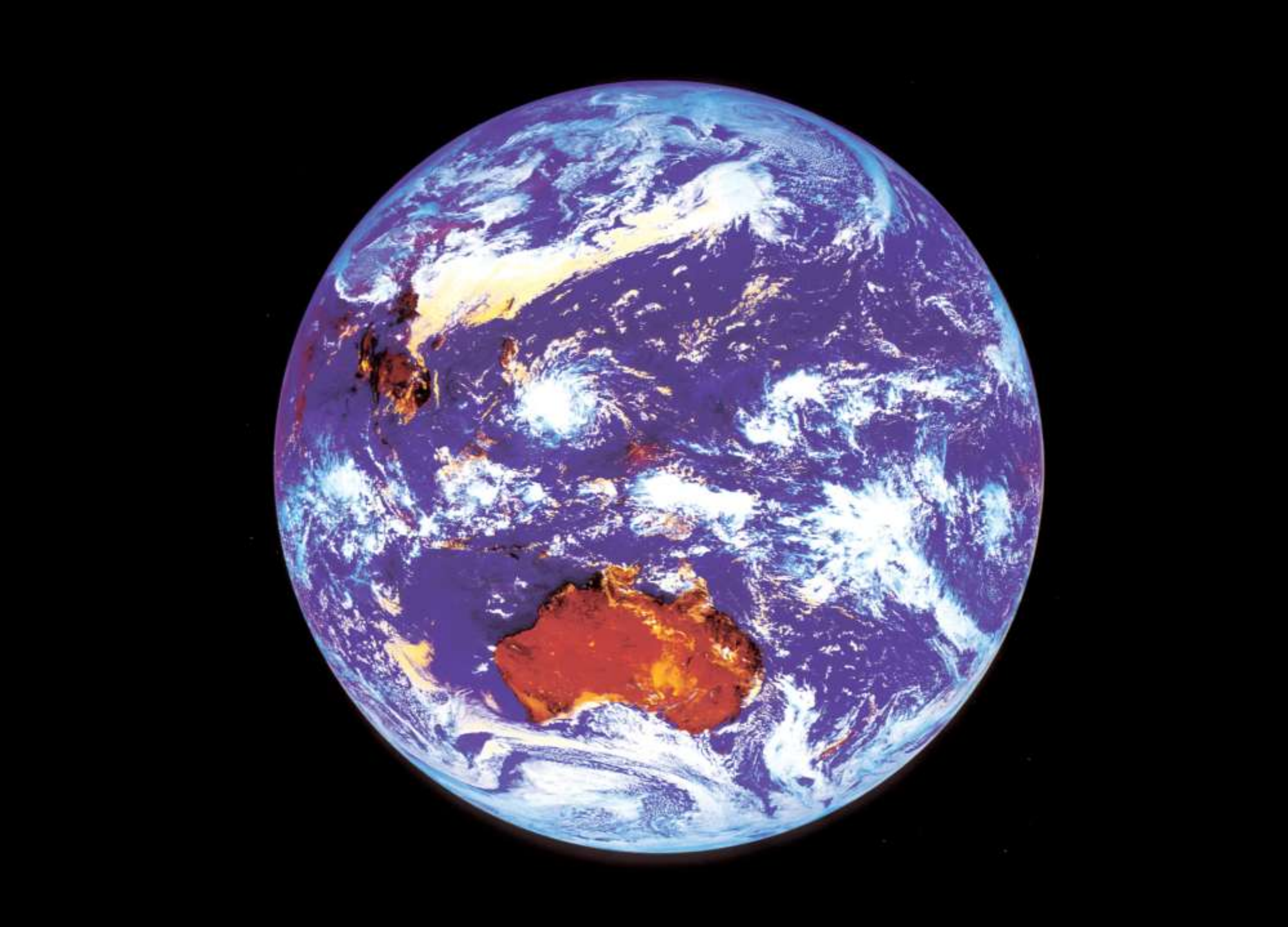
Attribution of past trend in sink rate (1959-2011)

- Contributions to observed trend in sink ($-0.75\%\ y^{-1}$ over 1959-2011)
  - Non-CO_2 forcing 12%
  - CO_2-temp coupling 10%
  - Other nonlinear 17%
  - Volcanic effects 25%
  - Non-exp emissions 36%
  - -------
  - 100%



(Some) answers: different for past and future

- PAST
  - Why are ratios among fluxes and stores (AF, CAF, *T*/*Q*) near constant from ~1850 to present, in the face of a 20-fold increase in emissions?
    - Because the carbon-climate system has been nearly LinExp: LinExp => exponential eigenmodes, constant ratios
  - To the extent that these ratios have changed, why so?
    - Growth rates 1959-2011: AF +0.3% y^{-1}, k_S −0.8% y^{-1}
    - 5 contributions: nonCO_2, C-T coupling, volcanoes, nonLin, nonExp
- FUTURE
  - How will AF, CAF and *T*/*Q* behave? Do we expect continuance of a near-proportional relationship between *T* and *Q*? If so, why?
    - Present near-LinExp behaviour will not continue
    - Near-constant T/Q will continue (~1.8 K EgC^{-1}; range 1.4 to 2.4)
    - nonCO_2 and C-T coupling will override nonExp emissions
    - BTW: chances of avoiding (2K, 3K) warming = (nil, slight)

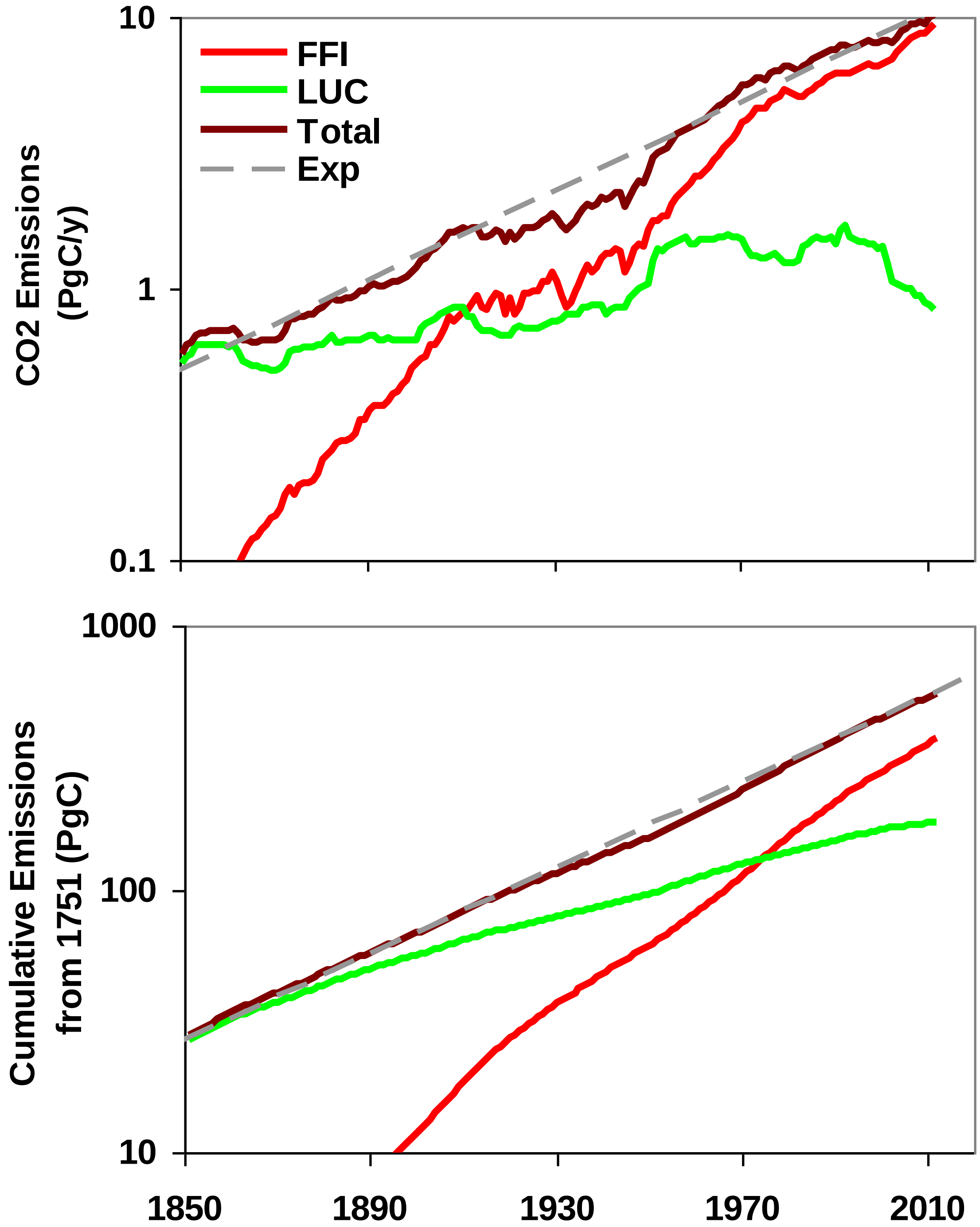


CO_2 emissions

- Total CO_2 emissions (FF + LUC) are growing nearly exponentially
  - FF acceleration
  - LUC slowdown

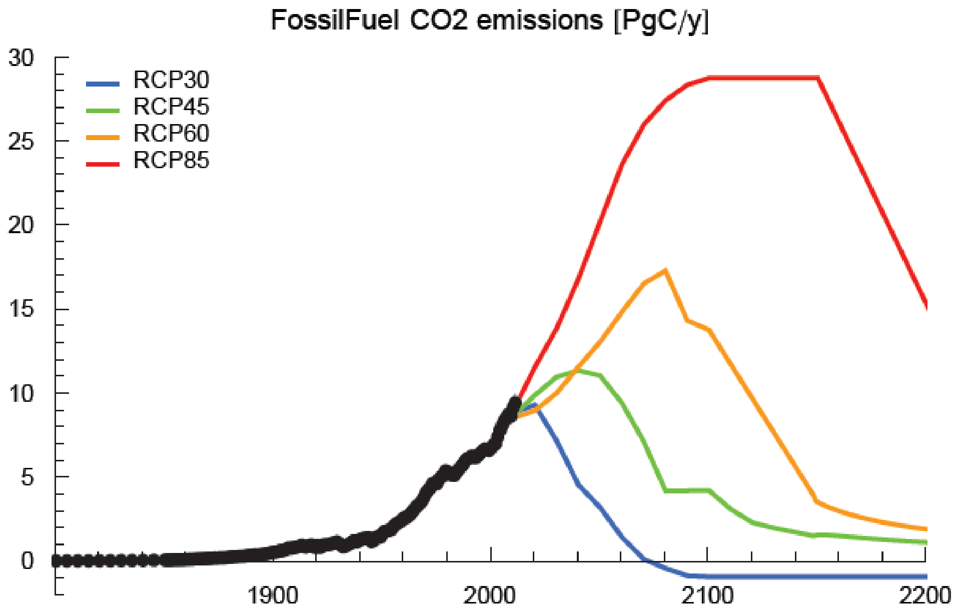
- Also, cumulative CO_2 emissions are growing nearly exponentially

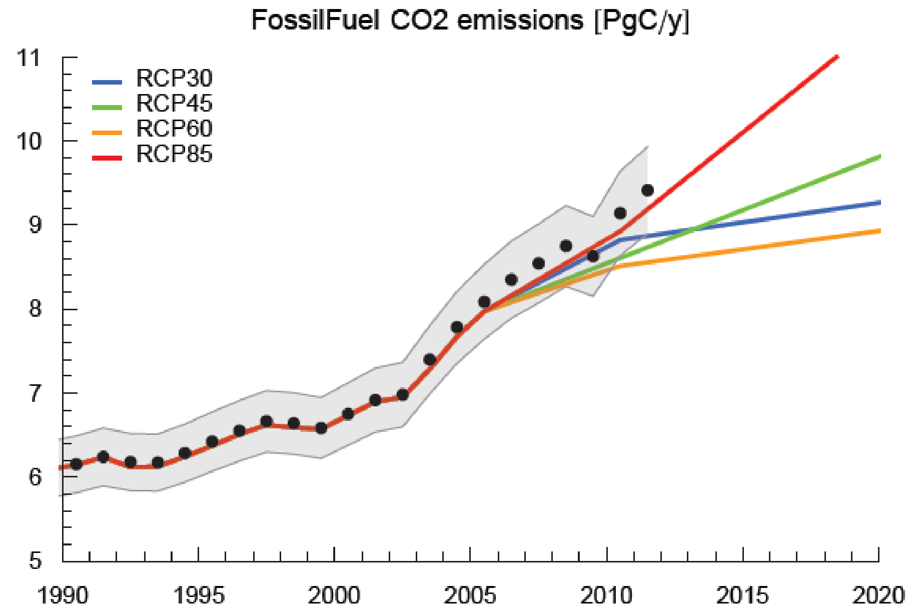
- For total CO_2 emissions:
  - Growth rate = 1.9%/y
  - Doubling time = 53 y



Global CO_2 emissions from fossil fuels

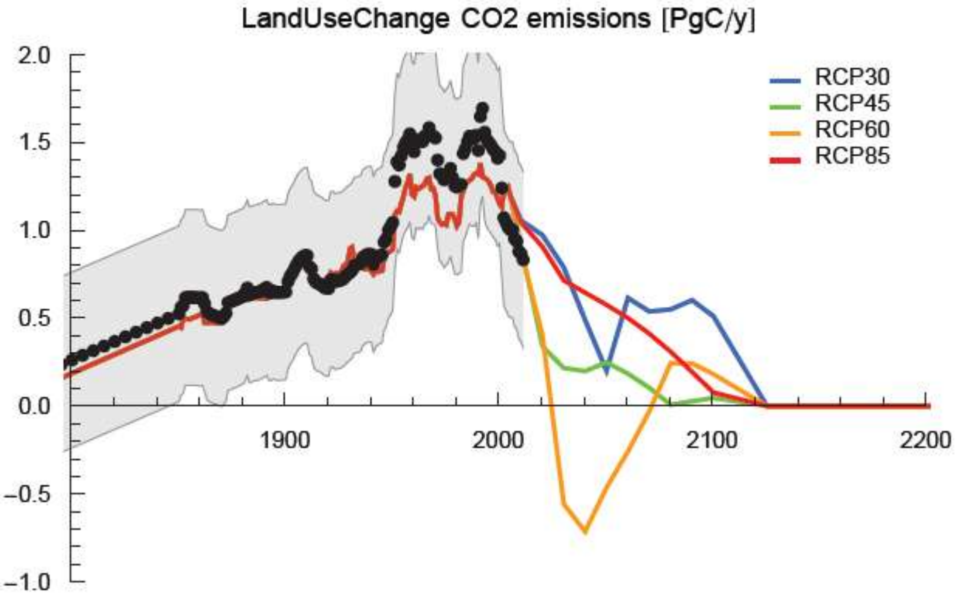
- Error band in plot: 1SD relative error = 0.055 (time independent)
- Assumed growth in FFI emissions from 2010 to 2011 = 3.0% (2001-2010 average)
  - (IEA gave 3.2% for FF only on 24-may-2012) (http://www.iea.org/newsroomandevents/news/2012/may/name,27216,en.html)

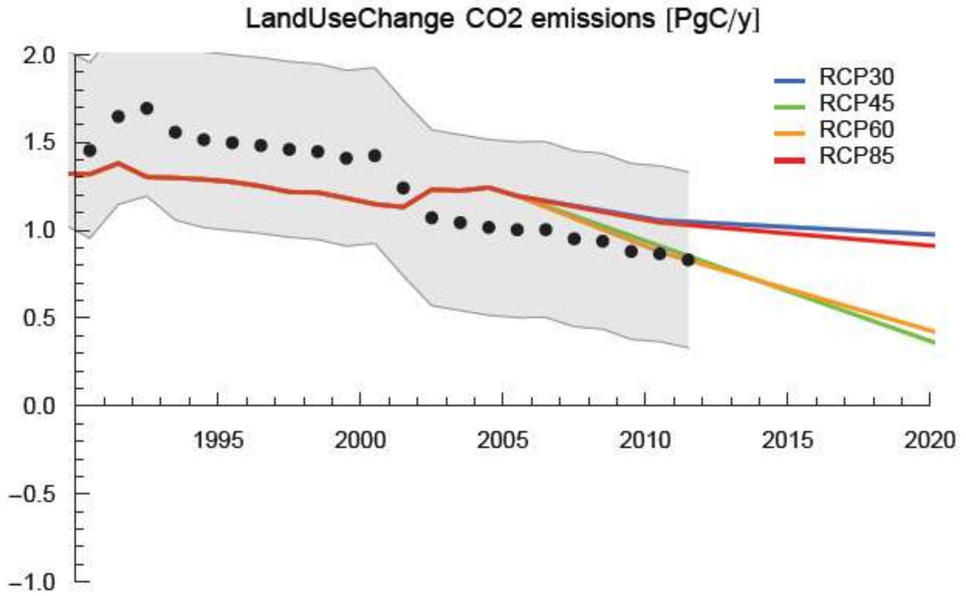




Global CO_2 emissions from Land Use Change

- Error band in plot: 1SD absolute error = 0.5 PgC/y (time independent)
- Assumed growth in LUC emissions from 2010 to 2011 = −4.0% (2001-2010 average)





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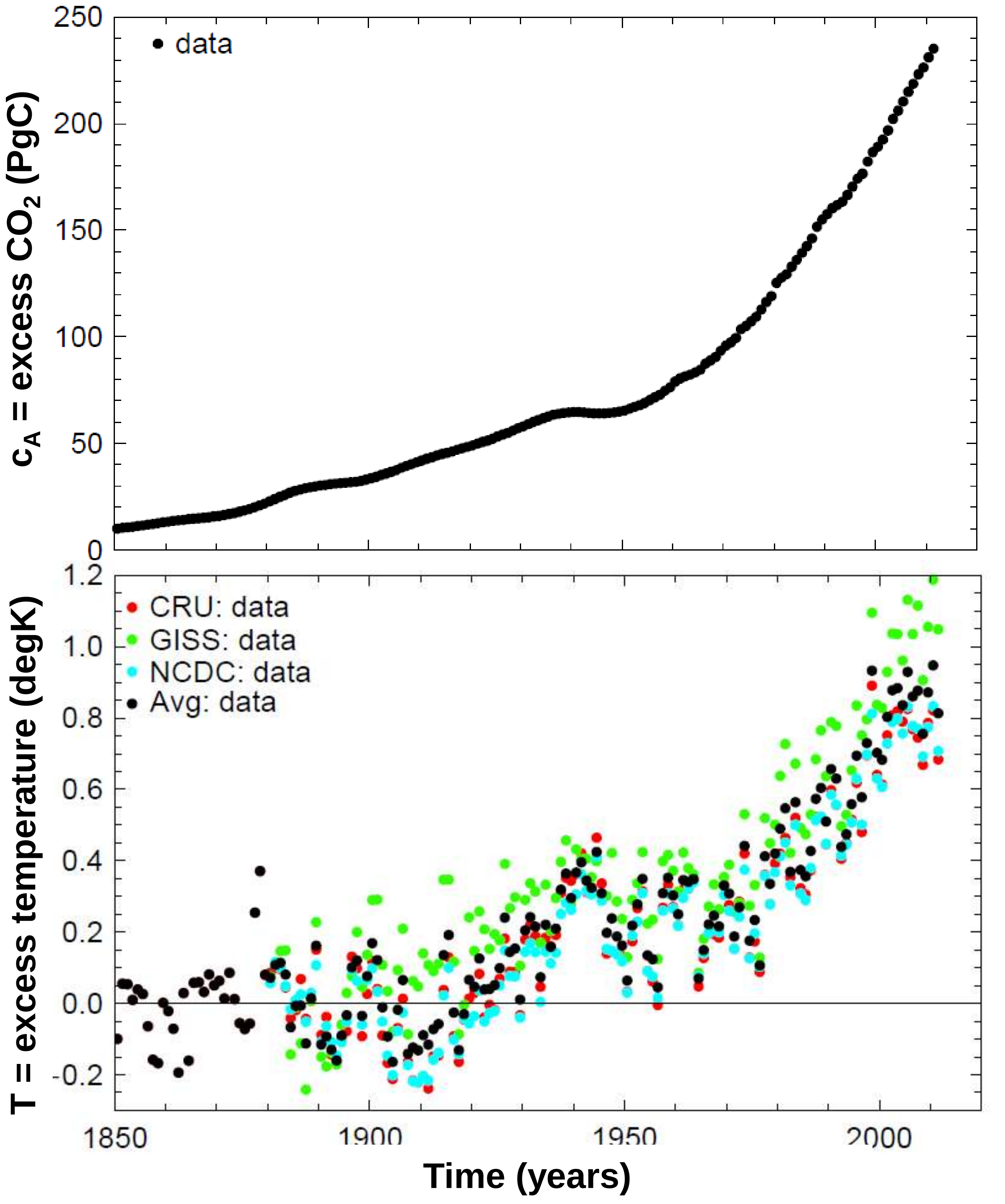
- Excess temperature = warming (ref 1880-1900)
- Plot against time



Linear time-invariant system: normal modes

- Linear system:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(t) - \mathbf{K}\mathbf{x}(t)\,; \qquad \mathbf{x}(0) = 0$$

with state variables $\mathbf{x}(t)$, forcing $\mathbf{f}(t)$, constant response matrix $\mathbf{K}$

- Eigenmodes of $\mathbf{K}$:

$$\mathbf{K}\mathbf{U} = \mathbf{U}\boldsymbol{\Lambda}\,, \qquad \mathbf{K} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^{-1}$$

$$\mathbf{U} = \left[\mathbf{u}^{(1)}, \mathbf{u}^{(2)}, \ldots\right]$$

matrix of column eigenvectors

$$\boldsymbol{\Lambda} = \mathrm{diag}\left(\lambda^{(1)}, \lambda^{(1)}, \ldots\right)$$

diagonal matrix of eigenvalues

- For a stable system, eigenvalues are negative for all modes m: $\lambda^{(m)} < 0$
- Transformed variables:

$$\mathbf{y}(t) = \mathbf{U}^{-1}\mathbf{x}(t)\,, \qquad \mathbf{x}(t) = \mathbf{U}\mathbf{y}(t)$$

- Diagonalised system of independent variables:

$$\frac{d\mathbf{y}(t)}{dt} = \mathbf{U}^{-1}\mathbf{f}(t) - \boldsymbol{\Lambda}\mathbf{y}(t)\,; \qquad \mathbf{y}(0) = 0$$

Linear time-invariant system: solution

- Solution: $$\mathbf{x}(t) = \int_0^t \mathbf{G}(t-\tau)\,\mathbf{f}(\tau)\,d\tau$$

  $\underbrace{\qquad\qquad}$ **Convolution of forcing with pulse response function (PRF)**

- Pulse Response Function (PRF):

$$\mathbf{G}(t) = \underbrace{\mathbf{Exp}(-\mathbf{K}t)}_{\text{Matrix exponential}} = \mathbf{U}\,\underbrace{\mathbf{Exp}(-\mathbf{\Lambda}t)}_{\text{Diagonal matrix}}\,\mathbf{U}^{-1}$$

- Elements of PRF matrix are sums of exponentials, each from a mode m
  - decay rates are eigenvalues, weight factors are given by eigenvectors

$$G_{ij}(t) = \sum_m a_{ij}^{m} \exp\left(-\lambda^{m} t\right), \qquad a_{ij}^{m} = \mathbf{U}_{im}\left(\mathbf{U}^{-1}\right)_{mj}$$

Linear time-invariant system with exponential forcing

- Forcing flux vector: $\mathbf{f}(t) = \left(f_0 \exp(r_1 t), 0, 0, \ldots\right)$

- Solution for pool i: $$x_i(t) = \underbrace{\sum_m \frac{a_{i1}^{(m)} f_0}{r_1 + \lambda^{(m)}} \Big[\exp(r_1 t)}_{\textbf{Eigenfunction}} \cancel{- \exp(-\lambda^{(m)} t)}\Big]$$

  - With exponential forcing, forcing and response have same shape: everything grows as $\exp(r_1 t)$

- => Theorem: for linear system (L) with exponential forcing (E):
  - All state variables grow at forcing rates (not response rates)
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SCCM results:
Progressively simplify model

- Model versions:
  1: Full model (FM)
  2: CO_2 only
  (remove non-CO_2 forcing = NC)
  3: Uncoupled
  (remove CC coupling = Co)
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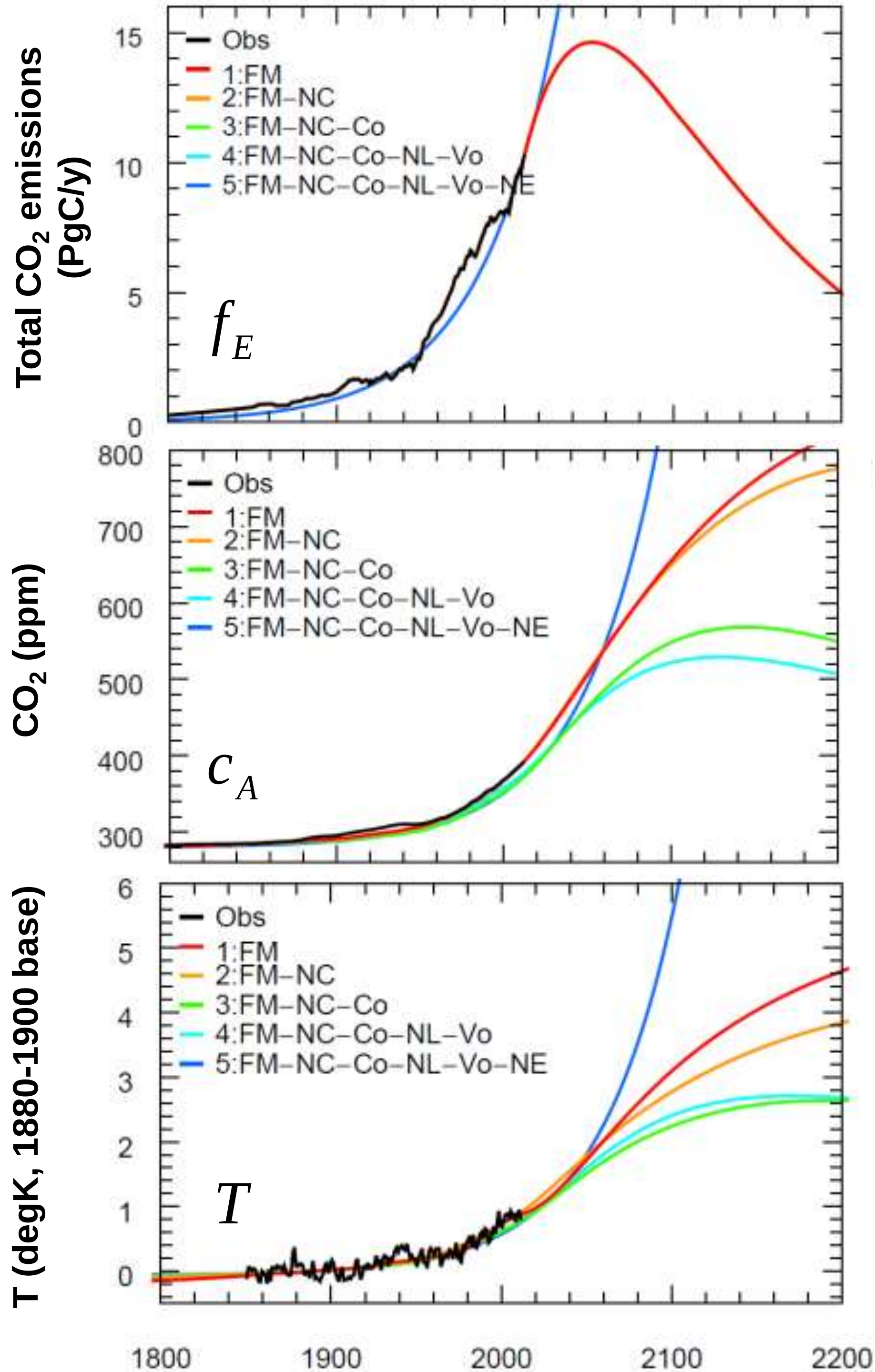
  (remove nonlinearities in CO_2 fluxes and radiative forcing)
  5: LinExp
  (impose exponential CO_2 emissions)



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